THE OPEN UNIVERSITY

TMA FORM (PT3)

STUDENT

2

SECTION 1 STUDENT TO COMPLETE

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19/7

Question Scores

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Overall Score
33	34	30																		97

30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70

TUTOR'S COMMENTS AND ADVICE TO STUDENT

Excellent

76

TMA S354 02 JOHN CLARK M4629534

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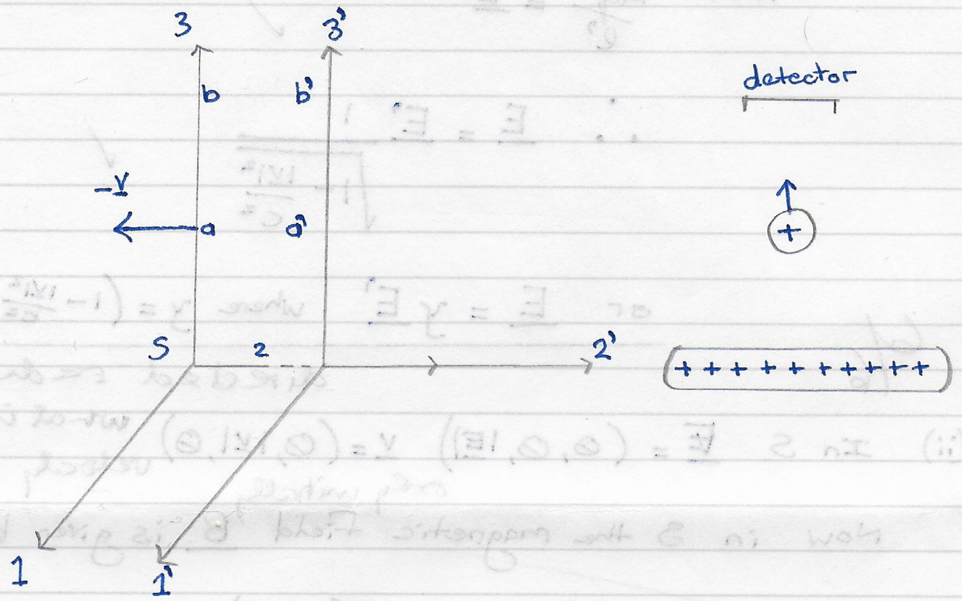


(i) In S' the electric field $\underline{E}'$ is proportional to the charge per unit length

i.e. $\underline{E}' \propto \frac{q'}{l'}$ or $\underline{E}' = k\frac{q'}{l'}$ where k = constant

Now by the PRINCIPLE OF RELATIVITY the electric field in S is also proportional to the charge per unit length in the same manner

i.e. $\underline{E} = k\frac{q}{l}$

BUT AS IN S THE ROD MOVES WITH VELOCITY $|\underline{V}|$ ALONG IT'S LENGTH - IT WILL BE RELATIVISTICALLY CONTRACTED BY A FACTOR GIVEN BY :-

$$l = l'\sqrt{1-\frac{|\underline{V}|^2}{c^2}}$$ where c = speed of light

Substituting for l above gives:-

$$\underline{E} = \frac{kq}{l'\sqrt{1-\frac{|\underline{V}|^2}{c^2}}}$$

As charge is LORENTZ INVARIANT ie. $q = q'$

we have $\underline{E} = k\frac{q'}{l'}\frac{1}{\sqrt{1-\frac{|\underline{V}|^2}{c^2}}}$

BUT $\frac{kq'}{e'^2} = \underline{E}'$

$$\therefore \quad \underline{E} = \underline{E}' \frac{1}{\sqrt{1 - \frac{|\underline{v}|^2}{c^2}}}$$

or $\underline{E} = \gamma \underline{E}'$ where $\gamma = \left(1 - \frac{|\underline{v}|^2}{c^2}\right)^{-1/2}$

excellent, 6/6

directed radially outwards

(ii) In S $\underline{E} = (0, 0, |\underline{E}|)$ $\underline{v} = (0, |\underline{v}|, 0)$ only initially

what is $\underline{v}$?, It is the velocity of the test charge

Now in S the magnetic field $\underline{B}$ is given by:-

In S' the test charge has a velocity component $\underline{v}$ along z axis, and a velocity component $\underline{v}_1$ (say) perpendicular to this axis. Initially $\underline{v}_1 = 0$.

$$\underline{B} = \varepsilon_0 \mu_0 (\underline{v} \times \underline{E})$$

$$\Rightarrow \underline{B} = \varepsilon_0 \mu_0 (v^2E^3 - v^3E^2, v^3E^1 - v^1E^3, v^1E^2 - v^2E^1)$$

$$\Rightarrow \underline{B} = \varepsilon_0 \mu_0 (|\underline{v}||\underline{E}| - 0, 0 - 0, 0 - 0)$$

$$\Rightarrow \underline{B} = (\varepsilon_0 \mu_0 |\underline{v}||\underline{E}|, 0, 0)$$

Using the LORENTZ FORCE LAW :-

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$$

$$\Rightarrow \underline{F} = q\left[(0, 0, |\underline{E}|) + (v^2B^3 - v^3B^2, v^3B^1 - v^1B^3, v^1B^2 - v^2B^1)\right]$$

$$\Rightarrow \underline{F} = q\left[(0, 0, |\underline{E}|) + (0, 0, -|\underline{v}|\varepsilon_0\mu_0|\underline{v}||\underline{E}|)\right]$$

$$\Rightarrow \underline{F} = (0, 0, (q|\underline{E}| - q|\underline{v}|^2|\underline{E}|\varepsilon_0\mu_0))$$

$$\Rightarrow \underline{F} = q|\underline{E}|(0, 0, (1 - |\underline{v}|^2\varepsilon_0\mu_0))$$

Now the speed of light c, is given by

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = \sqrt{\frac{1}{\varepsilon_0 \mu_0}}$$

$$\Rightarrow c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

$$\Rightarrow \varepsilon_0 \mu_0 = \frac{1}{c^2}$$

(ii)(cont) Substituting $\therefore \underline{F} = q|\underline{E}|\left(0, 0, \left(1 - \frac{|\underline{v}|^2}{c^2}\right)\right)$

Now magnitude is given by $\sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2}$

$$\therefore |\underline{F}| = \sqrt{\left[q|\underline{E}|\left(1 - \frac{|\underline{v}|^2}{c^2}\right)\right]^2}$$

$$\Rightarrow |\underline{E}| = \sqrt{(q|\underline{E}|)^2\left(1 - \frac{|\underline{v}|^2}{c^2}\right)^2}$$

$$\Rightarrow |\underline{F}| = q|\underline{E}|\sqrt{\left(1 - \frac{|\underline{v}|^2}{c^2}\right)\left(1 - \frac{|\underline{v}|^2}{c^2}\right)}$$

$$\Rightarrow |\underline{F}| = q|\underline{E}|\sqrt{1 - \frac{|\underline{v}|^2}{c^2}}\sqrt{1 - \frac{|\underline{v}|^2}{c^2}}$$

① or $|\underline{F}| = \frac{1}{\gamma^2} q|\underline{E}|$ where $\gamma = \left(1 - \frac{|\underline{v}|^2}{c^2}\right)^{-1/2}$

In S' There is no magnetic field as the charges are stationary

$\therefore \underline{F}' = q\underline{E}'$ (again assuming charge invariance)

BUT From (i) $\underline{E} = \gamma\underline{E}' \Rightarrow \underline{E}' = \frac{1}{\gamma}\underline{E}$

COMBINING WE HAVE

$$\underline{F}' = q\frac{1}{\gamma}\underline{E}$$

REMEMBERING THAT $\underline{E} = (0, 0, |\underline{E}|)$

$$\underline{F}' = \left(0, 0, \frac{1}{\gamma}q|\underline{E}|\right)$$

$$\Rightarrow |\underline{F}'| = \sqrt{\left(\frac{1}{\gamma}q|\underline{E}|\right)^2}$$

② $\Rightarrow |\underline{F}'| = \frac{1}{\gamma}q|\underline{E}|$

$\therefore$ COMBINING ① & ② gives:-

$$|\underline{F}| = |\underline{F}'|\frac{1}{\gamma}$$

$$\Rightarrow |\underline{F}'| = \gamma|\underline{F}|$$

This result is impossible in Newtonian mechanics, as Newtons second laws defines Force as the product of mass and acceleration (i.e. $\underline{F} = m\underline{a}$) and the

Correct But You have made rather heavy weather of it.

$\underline{F} = q\underline{E}(1 - v^2/c^2) = \frac{q'\gamma\underline{E}'}{\gamma^2} = \frac{\underline{E}'}{\gamma}$ because $q = q'$ charge is invariant, $\underline{E} = \gamma\underline{E}'$ from Part (i), $(1 - v^2/c^2) = \frac{1}{\gamma^2}$

$\therefore |\underline{F}| = \frac{|\underline{E}'|}{\gamma}$

Galilean transformations leave accelerations unaffected (ie $\underline{a}' = \underline{a}$)
Now inertial mass is also invariant (ie $m' = m$)
thus $|\underline{F}| = m|\underline{a}| = m'|\underline{a}'| = |\underline{F}'|$
Clearly, in Newtonian mechanics reference frames moving in uniform relative motion should agree on the magnitude of forces
ie $|\underline{F}| = |\underline{F}'|$ (Newtonian) ∴ ✓

8/10 9/9

Excellent.

(iii)

LORENTZ TRANSFORMATIONS

① $t' = \gamma\left(t - \frac{V}{c^2}x^1\right)$ *here $t' = \left(t - \frac{V}{c^2}x^2\right)$

② $x'^1 = \gamma(x^1 - Vt)$ *here $x'^2 = \gamma(x^2 - Vt)$

$x'^2 = x^2 \quad x'^3 = x^3$ $\qquad x'^1 = x^1 \quad x'^3 = x^3$

where $\gamma = \dfrac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$

Now $T = t_T - t_0$
and $T' = t'_T - t'_0$ $\qquad$ * V denotes $|\underline{V}|$

$$\therefore (t'_T - t'_0) = \left(\gamma t_T - \gamma\frac{V}{c^2}(x^2_T)\right) - \left(\gamma t_0 - \gamma\frac{V}{c^2}(x^2_0)\right)$$

$$\Rightarrow (t'_T - t'_0) = (\gamma t_T - \gamma t_0) - \left(\gamma\frac{V}{c^2}x^2_T - \gamma\frac{V}{c^2}(x^2_0)\right)$$

$$\Rightarrow (t'_T - t'_0) = \gamma(t_T - t_0) - \gamma\left(\frac{V}{c^2}(x^2_T) - \frac{V}{c^2}(x^2_0)\right)$$

AS S' moves with speed $|\underline{V}|$ relative to S
$(x^2) = |\underline{V}|t$ $\qquad$ (Vt in our notation)

$$\therefore (t'_T - t'_0) = \gamma\left[(t_T - t_0) - \left(\frac{V^2}{c^2}t_T - \frac{V^2}{c^2}t_0\right)\right]$$

$$\Rightarrow (t'_T - t'_0) = \gamma\left[(t_T - t_0) - \frac{V^2}{c^2}(t_T - t_0)\right]$$

$$\Rightarrow (t'_T - t'_0) = \gamma\left[(t_T - t_0)\left(1 - \frac{V^2}{c^2}\right)\right]$$

As $\gamma = \dfrac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$ and $\dfrac{\left(1 - \frac{V^2}{c^2}\right)}{\sqrt{1 - \frac{V^2}{c^2}}} = \sqrt{1 - \frac{V^2}{c^2}} = \dfrac{1}{\gamma}$

We have $(t'_T - t'_0) = \dfrac{1}{\gamma}(t_T - t_0)$ or $T' = \dfrac{1}{\gamma}T$

$$\Rightarrow (t_T - t_0) = \gamma(t'_T - t'_0) \text{ or } T = \gamma T'$$

(iii) (Cont)

Now the positions a, b and a', b' are all in the 3 direction and as the velocity of the boost is in the 2 direction the (x^3) component are unaffected by the LORENTZ TRANSFORMATIONS

$$\text{ie } x'^3 = x^3$$

$$\Rightarrow (x'^3_a) = (x^3_a) \text{ and } (x'^3_b) = (x^3_b)$$

$$\Rightarrow (b' - a') = (b - a) \checkmark$$

THUS COMPARING

$$m'_I = \frac{|E'|T'^2}{2(b'-a')} \quad \text{and} \quad m_I = \frac{|E|T^2}{2(b-a)}$$

Substituting $|E'| = \gamma|E|$ and $T' = \frac{1}{\gamma}T$ and $(b'-a') = (b-a)$

we have $m'_I = \dfrac{\gamma|E|T^2}{2(b-a)\gamma^2}$

$$\Rightarrow m'_I = \frac{1}{\gamma}\frac{|E|T^2}{2(b-a)}$$

But $\dfrac{|E|T^2}{2(b-a)} = m_I$

$$\therefore m'_I = \frac{1}{\gamma} m_I \quad \text{or} \quad m_I = \gamma m'_I \checkmark$$

m'_I is equated with the inertial mass of the particle in the rest frame S' ie it is the rest mass

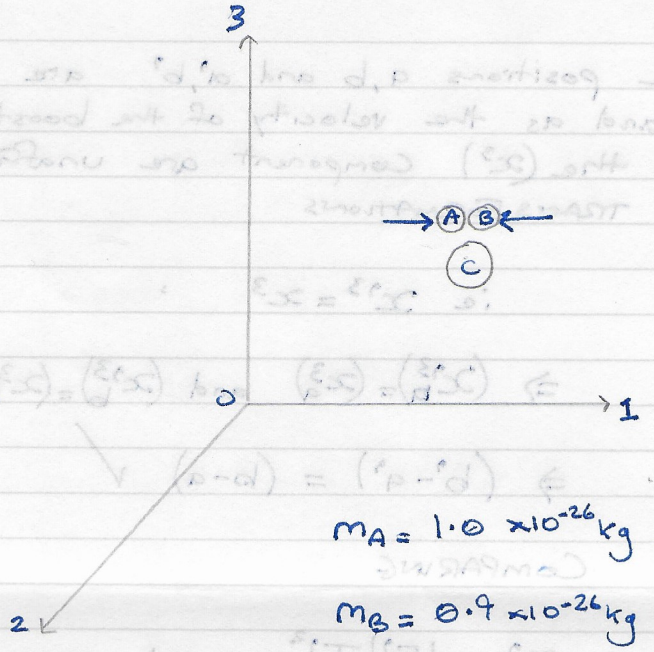
m_I can be equated with relativistic INERTIA, which does indeed increase with the particle's speed by a factor of $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

As shown above and in agreement with the maxim given ✓

Can't fault it.

10/10

2/



$m_A = 1.0 \times 10^{-26}\,kg \quad \underline{V_A} = \left(\tfrac{3}{4}c, 0, 0\right)$

$m_B = 0.9 \times 10^{-26}\,kg \quad \underline{V_B} = \left(-\tfrac{5}{6}c, 0, 0\right)$

$m_C = 1.9 \times 10^{-26}\,kg \quad \underline{V_C} = (0, 0, 0)$

(a) Newtonian mass is conserved ie:-

$$m_C = m_A + m_B$$

$$\Rightarrow 1.9 \times 10^{-26} = 1.0 \times 10^{-26} + 0.9 \times 10^{-26}\,kg$$

THIS IS IN AGREEMENT

Newtonian momentum $\underline{P} = m\underline{V}$ is conserved

$$\therefore \underline{P_C} = \underline{P_A} + \underline{P_B}$$

$$\Rightarrow 1.9 \times 10^{-26}(0,0,0) = \left(1.0 \times 10^{-26} \times \left(\tfrac{3}{4}c, 0, 0\right)\right) + \left(0.9 \times 10^{-26}\left(-\tfrac{5}{6}c, 0, 0\right)\right)$$

$$\Rightarrow (0,0,0) = \left(\tfrac{3}{4}c \times 10^{-26}, 0, 0\right) + \left(-\tfrac{4.5}{6}c \times 10^{-26}, 0, 0\right)$$

$$\Rightarrow (0,0,0) = \left(2.2485 \times 10^{-18}, 0, 0\right) + \left(-2.2485 \times 10^{-18}, 0, 0\right)$$

THIS IS ALSO IN AGREEMENT

<u>THEREFORE THE RESULTS ARE IN AGREEMENT WITH NEWTONIAN CONCEPTS</u> ✓

7/7

(b) IF the masses recorded are rest masses, the INERTIA of the particles is given by:-

$$m_I = m_{rest} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow m_{A_I} = \frac{1.0\times10^{-26}}{\sqrt{1-\frac{(3/4c)^2}{c^2}}} = \frac{1.0\times10^{-26}}{\sqrt{\frac{7}{16}}} = 1.51185\times10^{-26}\,kg$$

$$\Rightarrow m_{B_I} = \frac{0.9\times10^{-26}}{\sqrt{1-\frac{(-\frac{5}{6}c)^2}{c^2}}} = \frac{0.9\times10^{-26}}{\sqrt{\frac{11}{36}}} = 1.62816\times10^{-26}\,kg$$

NOW RELAVISTIC MOMENTUM $\underline{P} = m\gamma(v)\underline{V}$ WILL BE CONSERVED

$$\text{ie } \underline{P_c} = \underline{P_A} + \underline{P_B}$$

$$\Rightarrow m_c\gamma(v_c)\underline{V_c} = m_A\gamma(v_A)\underline{V_A} + m_B\gamma(v_B)\underline{V_B}$$

$$\Rightarrow m_c\gamma(v_c)\underline{V_c} = m_{A_I}\underline{V_A} + m_{B_I}\underline{V_B}$$

$$\Rightarrow m_c\gamma(v_c)\underline{V_c} = 1.51185\times10^{-26}\left(3/4c, 0, 0\right) + 1.62816\times10^{-26}\left(-5/6c, 0, 0\right)$$

$$\Rightarrow m_c\gamma(v_c)\underline{V_c} = \left(3.3993\times10^{-18}, 0, 0\right) + \left(-4.0676\times10^{-18}, 0, 0\right)$$

$$\Rightarrow m_c\gamma(v_c)\underline{V_c} = \left(-6.683\times10^{-19}, 0, 0\right) = m_{c_I}$$

Here inertial masses are conserved

This follows from consⁿ of relativistic energy. And you must say so explicitly.

$$\text{i.e. } m_c\gamma(v_c) = m_A\gamma(v_A) + m_B\gamma(v_B)$$

$$\Rightarrow m_c\gamma(v_c) = 1.51185\times10^{-26} + 1.62816\times10^{-26}$$

$$\Rightarrow m_c\gamma(v_c) = 3.14\times10^{-26}\,kg$$

$$\text{NOW } \underline{V_c} = \frac{m_{A_I}\underline{V_A} + m_{B_I}\underline{V_B}}{m_c\gamma(v_c)}$$

$$\Rightarrow \underline{V_c} = \frac{1}{3.14\times10^{-26}}\left(-6.683\times10^{-19}, 0, 0\right)$$

$$\Rightarrow \underline{V_c} = \left(-2.12834\times10^{7}, 0, 0\right)\,ms^{-1}$$

6/7

(b) (cont)

Now $m_c\gamma(v_c) = m_c \dfrac{1}{\sqrt{1-\frac{|v_c|^2}{c^2}}} = m_{c_I}$

$$\Rightarrow m_c = m_{c_I}\sqrt{1-\frac{|v_c|^2}{c^2}}$$

$$\Rightarrow m_c = 3.14\times10^{-26}\sqrt{1-\frac{(-2.12834\times10^7)^2}{c^2}}$$

$$\Rightarrow \underline{m_c = 3.132\times10^{-26}\ \text{kg}}$$ ✓

$13\frac{1}{2}/14$

(c)

In the Centre of mass (C of M) Frame the original frame has a boost velocity (V_b) of $+2.12834\times10^7$ m s^{-1}

No

do not forget to state units

We have the velocities in the moving frame (U')
We require their equivalents in the static C of M frame (U)

In the lab frame particle c moves along -ve 1 axis with a speed of 2.13×10^7 m s^{-1}. It will be at rest in a frame which moves with the same velocity relative to the lab frame.

Now $U' = \dfrac{U'' + V_b}{1+\frac{V_b}{c^2}U''}$

$$\Rightarrow U'_A = \frac{3/4c + 2.12834\times10^7}{1+\frac{2.12834\times10^7\times 3/4c}{c^2}}$$

$$\Rightarrow U'_A = 2.33685\times10^8$$

$$\therefore \underline{U_A} = (2.33685\times10^8, 0, 0)$$

$$\Rightarrow \underline{|\underline{U_A}| = 2.33685\times10^8\ \text{ms}^{-1}}$$ ✓

Similarly $U'_B = \dfrac{-5/6c + 2.12834\times10^7}{1+\frac{2.12834\times10^7\times -5/6c}{c^2}}$

$$\Rightarrow U'_B = -2.42915\times10^8$$

$$\Rightarrow \underline{U_B} = (-2.42915\times10^8, 0, 0)$$

$$\Rightarrow \underline{|\underline{U_B}| = -2.42915\times10^8\ \text{ms}^{-1}}$$ ✓

(d) In the CofM Frame again

$$\underline{P} = m\gamma(v)\underline{V} \quad \text{where } \gamma(v) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow \underline{P_{A_{cm}}} = \frac{1.0\times10^{-26}}{\sqrt{1-\frac{(2.33685\times10^8)^2}{c^2}}}\left(2.33685\times10^8, 0, 0\right)$$

$$\Rightarrow \underline{P_{A_{cm}}} = \left(3.7305\times10^{-18}, 0, 0\right)$$ ✓

$$\underline{P_{B_{cm}}} = \frac{0.9\times10^{-26}}{\sqrt{1-\frac{(-2.42915\times10^8)^2}{c^2}}}\left(-2.42915\times10^8, 0, 0\right)$$

$$\Rightarrow \underline{P_{B_{cm}}} = \left(-3.7305\times10^{-18}, 0, 0\right)$$ ✓

$$\text{i.e. } \underline{P_{B_{cm}}} = -\left(\underline{P_{A_{cm}}}\right)$$ ✓

7/7 Excellent.

Q3 It is, of course, very true that Special Relativity predicts that observers in relative motion will disagree about many factors relating commonly observed events. However the theory also leaves many other qualities unaffected by the observers relative velocity.

Firstly and foremostly, the two basic postulates of Special Relativity assert that the PRINCIPLE OF RELATIVITY is upheld and that the speed of light (in free space) has the same value for all inertial observers. So it is clear that an event, for instance the collision of two particles, will be described using the laws of physics by all inertial observers, although the values they assign to quantities may vary. It is also clear that if two events are used to measure the speed of light by passing rays between them, all inertial observers will measure the same value, c. ✓

Other quantities are also inherently unaffected by the relative velocity of observers (or are LORENTZ

INVARIANT), for instance the charge of a particle and its rest mass (as opposed to INERTIA).

Moreover it is clear from the LORENTZ TRANSFORMATIONS of positions at right angles to the direction of velocity (ie $x'^2 = x^2$ & $x'^3 = x^3$) that these quantities and therefore lengths measured in these planes (provided both points separated by the length are located simultaneously) are Lorentz invariant.

The INVARIANT INTERVAL squared between two events (a & b) in four dimensional Minkowski space-time is also a Lorentz invariant.

$$(S_{ab})^2 = (ct_b - ct_a)^2 - (x^1_b - x^1_a)^2 - (x^2_b - x^2_a)^2 - (x^3_b - x^3_a)^2$$

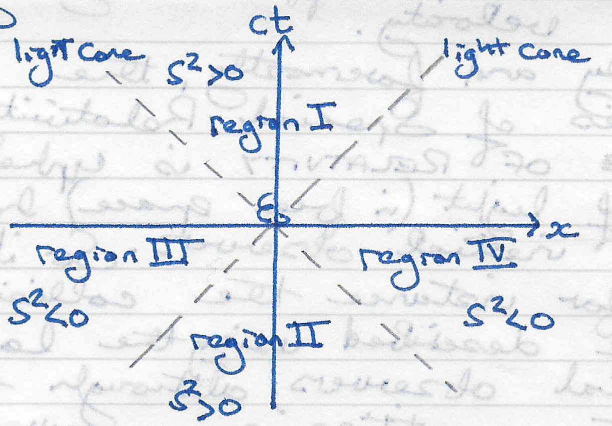
$$\text{ie} = (S'_{ab})^2 = (ct'_b - ct'_a)^2 - (x'^1_b - x'^1_a)^2 - (x'^2_b - x'^2_a)^2 - (x'^3_b - x'^3_a)^2$$

Following this point it can be stated that if two events are joined by a light pulse

ie if $(ct_b - ct_a)^2 - (x_b - x_a)^2 = 0$ (2 dimensional)

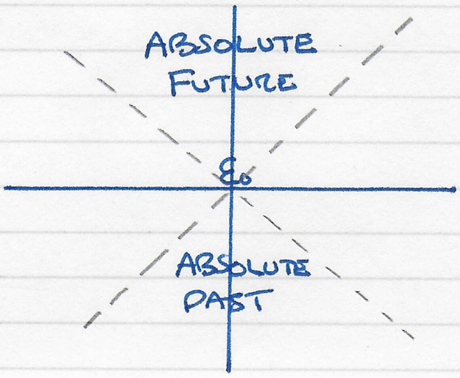
then they will be so joined for all inertial observers. Similarly, if $(S_{ab})^2$ is positive and the events are linked by a time-like interval, they will be for all inertial observers: as is the case if $(S_{ab})^2$ is negative and the interval is space-like.

Indeed events related to an event at the origin of the following diagram which fall in areas I II III and IV, will occur in these regions for all inertial observers:—



This fact, that the Lorentz transformations and invariant quality of $(S_{ab})^2$ prevent events from "crossing the light cone" means that events in region I will be in the ABSOLUTE FUTURE to an event at the origin for all inertial

3/(Cont) observers and similarly all events in region II will be in the ABSOLUTE PAST to an event at the origin for all inertial observers.



(It should also be remembered that the simultaneity of events not separated in space can also be agreed by all inertial observers.)

This Lorentz invariant quality of events in the absolute future means that all inertial observers in relative uniform motion, may disagree on the shapes of world lines of events, but they will agree on the time order of the events on the world line (i.e. its "direction")

In this frame it is clear by its definition that proper time (τ) is Lorentz invariant; this follows from its mechanical construction as a clock carried on the world line and also by the equation giving τ :-

$$\Delta\tau = \frac{\Delta S}{c}$$

As S and c are Lorentz invariant - τ must also be

Finally it can also be shown that the time and linear components of relativistic momentum (or the momentum pair) p^0 and p^1 can be combined in a quantity, similar to $(S_{ab})^2$, that is also Lorentz invariant

i.e. $(p^0)^2 - (p^1)^2 = m^2c^2 = (p'^0)^2 - (p'^1)^2$

Excellent, rather long

30/30.